# PHYS 320 ANALYTICAL MECHANICS

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Fall 2108

Apollo launch

## TODAY

Vector review!

Newton's Laws

## Vectors: scalar (dot) product

$$\vec{A}\cdot\vec{B} = (A_x B_x) + (A_y B_y) + (A_z B_z)$$

or

$$\vec{A}\cdot\vec{B} = \sum_{i,j=1}^{3} A_i B_j \hat{e}_i \cdot \hat{e}_j = \sum_{i=1}^{3} A_i B_i$$

$\delta_{ij}$ = Kronecker delta

**commutative and associative!**

**Also** $\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta$

## Vectors: scalar (dot) product

- Perform the operations with the vectors given

$$\vec{A} = 3\hat{i} + 2\hat{j} + 4\hat{k}$$

$$\vec{B} = -2\hat{i} + 5\hat{j} - \hat{k}$$

$$\vec{C} = xz\hat{i} - 3y\hat{j} - xy\hat{k}$$

$i)$ $\vec{A}\cdot\vec{B} = ?$

$ii)$ $\vec{A}\cdot\vec{C} = ?$

$iii)$ What is the angle between $\vec{A}$ and $\vec{B}$?

# Vectors: vector (cross) product

$$\vec{A}\times\vec{B} = \hat{i}(A_yB_z - A_zB_y) + \hat{j}(A_zB_x - A_xB_z) + \hat{k}(A_xB_y - A_yB_x)$$

or

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

determinant

**distributive, but not commutative**

$\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$

**Also** $\vec{A}\times\vec{B} = |\vec{A}||\vec{B}|\sin\theta\,\hat{n}$

direction determined by RH Rule

# Vectors: vector (cross) product

- Perform the operations with the vectors given

$$\vec{A} = 3\hat{i} + 2\hat{j} + 4\hat{k}$$

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$i)$ $\vec{A}\times\vec{B} = ?$

$ii)$ $\vec{A}\times\vec{C} = ?$

$iii)$ What is the angle between $\vec{A}$ and $\vec{B}$?

# Cross Product: Another form?

- For the scalar product, $\vec{A}\cdot\vec{B}=\sum_{i=1}^{3}A_iB_i$ is so cute and compact!
- For vector product, we can write that if

$\vec{C}=\vec{A}\times\vec{B}$ then $C_i=\sum_{j,k=1}^{3}\varepsilon_{ijk}A_jB_k$

where the Levi-Civita symbol (pseudo-tensor) is

$$\varepsilon_{ijk}=\begin{cases}+1 & \text{if i, j, k form an even permutation of 1,2,3}\\ 0 & \text{if any index} = \text{any other index}\\ -1 & \text{if i, j, k form an odd permutation of 1,2,3}\end{cases}$$

# Levi-Civita (permutation) symbol

- In other words …

$\varepsilon_{123} = \varepsilon_{231} = \varepsilon_{312} = +1$

$\varepsilon_{113} = \varepsilon_{232} = \varepsilon_{221} = 0$

$\varepsilon_{132} = \varepsilon_{213} = \varepsilon_{321} = -1$

- A useful property of the Levi-Civita symbol:

$$\sum_{k=1}^{3}\varepsilon_{ijk}\varepsilon_{lmk}=\delta_{il}\delta_{jm}-\delta_{im}\delta_{jl}$$

# Vectors: triple vector products

- There are several (see text)
- Most likely to use:

volume of parallelepiped

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = \vec{B}\cdot(\vec{C}\times\vec{A}) = \vec{C}\cdot(\vec{A}\times\vec{B})$$

$$\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$$